MATHEMATICS (GRADES K-12)

2013

Mathematics
Curriculum Guide

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Philosophy Statement

Morrison Academy strives to develop students who obtain a mastery of, and appreciation for, the content and processes of mathematics, as well as an understanding of how mathematics relates to other domains in life. While the study of mathematics occurs mostly within finite creation, it points us towards the character of an infinite God who values logic, order and consistency. Therefore, the purpose of mathematical instruction at Morrison is to develop students whose mastery of content includes insight into logical processes that will enable students to make meaningful connections to a world of order and help them transfer math concepts to other domains consistent to life. Such tangible engagement with math will help Morrison's students dynamically impact their world for Christ.

Vision for Our Mathematic Learners

Moral and Ethical Citizen

1. Students will encourage and help one another to work to the best of their ability in math.
2. Students will use their mistakes (failures) as a learning experience to improve their performance on future tasks.

Spiritual Discerner

1. Students will be able to discern evidence of God's structure of creation in math sequences and patterns. (CCSS.MP.1)[1] (CCSS.MP.7)
2. When exposed to a non-biblical worldview relating to mathematical concepts, students will be able to present a defense for the Biblical alternative. (CCSS.MP.3)

Critical and Creative Thinker

1. Students will analyze, interpret, evaluate, and synthesize mathematical concepts. (CCSS.MP.3)
2. Students will utilize mathematics to problem-solve effectively. (CCSS.MP.1) (CCSS.MP.3)
3. Students will be able to apply the skills of logic and precision in drawing conclusions to life issues and problems. (CCSS.MP.3) (CCSS.MP.4) (CCSS.MP.6)

[1]CCSS.MP.1 refers to the Common Core Mathematical Practice #1. Please see Appendix D for more details.

Life Long Learner

1. Students will utilize technology appropriately to apply higher order thinking concepts. (CCSS.MP.5)
2. Students will integrate biblical principles and problem solving skills in life decisions. (CCSS.MP.1)
3. Students will persevere in solving mathematical problems and apply the same discipline to other areas of life. (CCSS.MP.1)
4. Students will be able to apply the complementary abilities of reasoning abstractly and reasoning quantitatively to solve mathematical and life problems. (CCSS.MP.2) (CCSS.MP.4)

Effective Communicator

1. Students will be able to construct persuasive narratives that show a valid, logical progression. (CCSS.MP.3)
2. Students will communicate mathematical reasoning skillfully and purposefully. (CCSS.MP.6)
3. Students will understand how mathematics helps people communicate ideas, patterns, solutions, and reasoning to others. (CCSS.MP.3) (CCSS.MP.7) (CCSS.MP.8)

Wise and Responsible Steward

1. Students recognize the value of learning mathematics.
2. Students can apply mathematical concepts to resolve life issues. (CCSS.MP.4)

3. Students will be able to apply principles of mathematics and logic to analyze and draw conclusions about quality of life issues such as environmental concerns. (CCSS.MP.4)

Hallmarks

Assessment

The main goal of assessment at Morrison is to enable students to meet or exceed learning targets within their God given ability. To achieve this, a balance of formative assessments (informing students of their progress) and summative assessments (measuring attainment of learning targets at a given point in time) are used. Formative assessments are given on an ongoing basis to inform teachers and students about adjustments that are needed in teaching and learning. These assessments can be formal or informal and should allow students sufficient time to refine their understanding (William, Thompson, 2007). Summative assessments give teachers, students and parent a summary of how well the student has grasped the content. Unit assessments (assessing a group of related learning targets), Common Assessments (using the same assessment for all students in the Morrison system) and external assessments all form part of summative assessments.

William, Dylan, and Marnie Thompson. "Integrating Assessment with Instruction: What Will It Take to Make It Work?" In The Future of Assessment: Shaping Teaching and Learning, edited by Carol A. Dwyer. Mahwah, N.J.: Lawrence Erlbaum Associates, 2007.

Biblical Worldview Integration

One of Morrison's visions is to prepare our students to "impact their world dynamically for Christ." In order to accomplish this, Morrison teachers help our students learn to think Biblically and critically about all subjects and all areas of their lives. Morrison math teachers incorporate this goal in their instruction and relationships with students through "content integration, classroom practice integration, spontaneous integration and mentoring." (Procedure 130) Teachers exemplify Christ-like behavior through classroom practice and mentoring, and demonstrate integration of Biblical concepts in their interpersonal relations with students and staff. They also purposefully draw attention to Biblical connections in math content. Meaningful connections should highlight principles that build faith, help in living the Christian life, and help students integrate their faith and biblical principles across all subjects and all areas of their lives. Teachers take advantage of opportunities to help students understand the difference between the Biblical worldview and other worldviews, religions and philosophies. In math, we see the order and design of Creator God and the nature of God Himself as a God of detail, logic, order. We realize that God created mathematical concepts and man is in the process of discovering what God has designed.

http://www.barna.org/barna-update/article/5-barna-update/131-a-biblical-worldview-has-a-radical-effect-on-a-persons-life
http://bible.biblicalintegration.com/what-biblical-integration-is-and-isnt/

Technology

Technology at Morrison enhances the mathematics curriculum and provides a more comprehensive learning environment for students. Technology tools include computers (with appropriate software), calculators, manipulatives, and interactive boards. Technology enables users to explore topics in more depth and in more interactive ways. Activities that take advantage of the capabilities of technology can

extend beyond or significantly enhance what can be done without technology, leading to experiences that go beyond the classroom. Using technology creates multiple methods of entry into the curriculum, and allows universal access for all learners.

CAST (2011). Universal design for learning guidelines version 2.0. Wakefield, MA.

Common Core

Morrison Academy uses the Common Core State Standards for Mathematics as the basis for our revised math curriculum. These academically rigorous standards have been adopted by nearly all the 50 U.S. States. While Morrison has always used an academically rigorous curriculum, having greater continuity between our math curriculum and the curricula used in the U.S. will benefit Morrison students that move between Taiwan and the U.S. Access to better-aligned resources, supporting materials, and assessments will also continue to make the Morrison mathematics program excellent. The Common Core Mission Statement is as follows: "The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy."

http://www.corestandards.org
http://www.aft.org/pdfs/americaneducator/summer2002/curriculum.pdf

Higher Order Thinking Skills/Problem Solving

Recognizing that students must become proficient in computational and procedural skills, Morrison also emphasizes the value of students improving their higher order thinking skills. Higher order thinking skills can be defined in terms of transfer, critical thinking, and problem solving. A student that has developed higher order thinking skills is able to use what he has learned. The student reasons, questions, investigates, observes, describes, compares, connects, finds complexity, and explores other viewpoints. Also, the student learns with understanding, critically evaluates, formulates creative alternatives, and communicates effectively. By teaching and cultivating these thought patterns, Morrison helps students come to see mathematics as a way of finding solutions to problems that occur outside the classroom, rather than a given set of algorithms to be applied to problems in the textbook. Thus, students grow in their ability and persistence in problem solving through experience in solving problems at a variety of levels of difficulty and at every level in their mathematical development.

How to Assess Higher-Order Thinking Skills in Your Classroom (Susan M.Brookhart 2010, ASCD)
http://www.ascd.org/publications/books/109111/chapters/Introduction.aspx

Quality of Instruction

Research has shown that instruction is more effective when it's personalized and teachers understand learners' interests, strengths, and contributions to the class. Morrison provides a framework for comprehending and integrating new information with old by tapping into the students' prior knowledge, experiences, and interests (Wiggins, McTighe, 2006). Teachers utilize research-based instructional strategies and assessment to plan and implement their instruction. They develop life-long learners and rational and critical thinkers, thus teaching for meaning rather than just procedural answers. They lead class discussions with warranted open-ended questions, and provide choices when appropriate.

(www.cesdp.nmhu.edu--Connections in Reading and Mathematics Instruction, Center for the Education and Study of Diverse Populations)

Engaged Instructional Time Requirements

(Refer to Procedure 295)

Strands: Kindergarten

Counting and Cardinality
Operations and Algebraic Thinking
Number and Operations in Base Ten
Measurement and Data
Geometry

Strands: First and Second Grade

Operations and Algebraic Thinking
Number and Operations in Base Ten
Measurement and Data
Geometry

Strands: Third through Fifth Grade

Operations and Algebraic Thinking
Number and Operations in Base Ten
Number and Operations—Fractions
Measurement and Data
Geometry

Strands: Sixth and Seventh Grade

Ratios and Proportional Relationships
The Number System
Expressions and Equations
Geometry
Statistics and Probability

Strands: Eighth Grade

The Number System
Expressions and Equations
Functions
Geometry
Statistics and Probability

Strands: Algebra, Geometry and Algebra II

Number and Quantity
- The Real Number System
- Quantities
- The Complex Number System
- Vector and Matrix Quantities

Algebra
- Seeing Structure in Expressions
- Arithmetic with Polynomials and Rational
- Expressions
- Creating Equations
- Reasoning with Equations and Inequalities

Functions
- Interpreting Functions
- Building Function
- Linear, Quadratic, and Exponential Models
- Trigonometric Functions

Modeling

Geometry
- Congruence
- Similarity, Right Triangles, and Trigonometry
- Circles
- Expressing Geometric Properties with Equations
- Geometric Measurement and Dimension
- Modeling with Geometry

Statistics and Probability
- Interpreting Categorical and Quantitative Data
- Making Inferences and Justifying Conclusions
- Conditional Probability and the Rules of Probability
- Using Probability to Make Decisions

Strands: Pre-Calculus

- Trigonometry
- Linear Algebra
- Discrete Mathematics
- Analytic Geometry
- Elementary Functions
- Limits

Strands: AP Calculus Strands

- Functions, Graphs, and Limits
- Derivatives
- Integrals
- Polynomial Approximations and Series (Calculus BC only)

Strands: AP Statistics

- Exploring Data
- Sampling and Experimentation
- Anticipating Patterns
- Statistical Inference

Scope and Sequence for Kindergarten through Fifth Grade

Strands	Grade Levels					
	K	1st	2nd	3rd	4th	5th
Counting & Cardinality						
Operations & Algebraic Thinking						
Number & Operations in Base Ten						
Number & Operations - Fractions						
Measurement & Data						
Geometry						

Scope and Sequence for Sixth through Eighth Grade

Strands	Grade Levels		
	6	7	8
Ratios and Proportional Relationships			
The Number System			
Expressions and Equations			
Geometry			
Statistics and Probability			
Functions			

Scope and Sequence for Ninth through Twelfth Grade

Course Name	***Grade Level***				
	Eight	***Nine***	***Ten***	***Eleven***	***Twelve***
Algebra 1	***When approved***				
Geometry					
Algebra 2					
Pre-Calculus					
AP Statistics					
AP Calculus					

Resources for Instruction

Elementary (K-5)

Primary Resources

My Math (McGraw Hill) - consumable

	Student Ed. + 6 yrs Online eStudent Ed.	Teacher Ed. + 6 yrs Online eTeacher Ed.	Classroom Manipulative Kit	Assessment Master
K	9780021170685	9780021170562	9780021064816	9780021064816
1st	9780021170692	9780021170579	9780021064816	9780021064816
2nd	9780021170708	9780021170586	9780021064816	9780021064816
3rd	9780021170715	9780021170593	9780021064816	9780021064816
4th	9780021170722	9780021170609	9780021064816	9780021064816
5th	9780021170739	9780021170616	9780021064816	9780021064816

Secondary (6-12)

Primary Resources

Holt McDougal Larson

	Student Ed.	Teacher Ed.	E-Pub	One-Stop Teacher Guide
6	978-0-547-64716-6	978-0-547-64721-0	978-0-547-77593-7	978-0-547-68817-6
7	978-0-547-64717-3	978-0-547-64724-1	978-0-547-77594-4	978-0-547-68827-5
8	978-0-547-64719-7	978-0-547-64727-2	978-0-547-77595-1	978-0-547-68825-1
Algebra	978-0-547-64713-5	978-0-547-64706-7	978-0-547-77599-9	978-0-547-71056-3
Geometry	978-0-547-64714-2	978-0-547-64708-1	978-0-547-77600-2	978-0-547-71081-5
Algebra II	978-0-547-64715-9	978-0-547-64711-1	978-0-547-64711-1	978-0-547-71088-4

Course (Publisher Year Author Edition)	Student Ed.	Teacher Ed.
Pre-Calculus (Houghton Mifflin 2005 Larson 4th Ed) ***Will order new books in 2014*	0-618-39478-8	0-618-39479-6
AP Statistics (Pearson 2010 Bock 3rd Ed)	978-0-321-57044-4	978-0-321-57103-8 (videos on DVD) 978-0-321-57094-9 (graphing calculator manual)
AP Calculus (Cengage 2010 Larson 9th Ed)	978-0547167022	

Pacing, Sequence and Course Offerings:

Accelerated Math Track (beginning in Grade 8)
Seventh grade students who earn a 90% or higher in seventh grade math, pass the Algebra placement test (85% or greater), and have demonstrated good study habits in grade 7 math will have the option of taking Algebra 1 in their 8th grade year. *(See 8th grade Algebra I student entrance requirements on page 14 for more information)*. Students in this track are required to score a B- or higher both in their Algebra I course and on the Algebra Exit Exam, or they may be required to repeat Algebra I in 9th grade

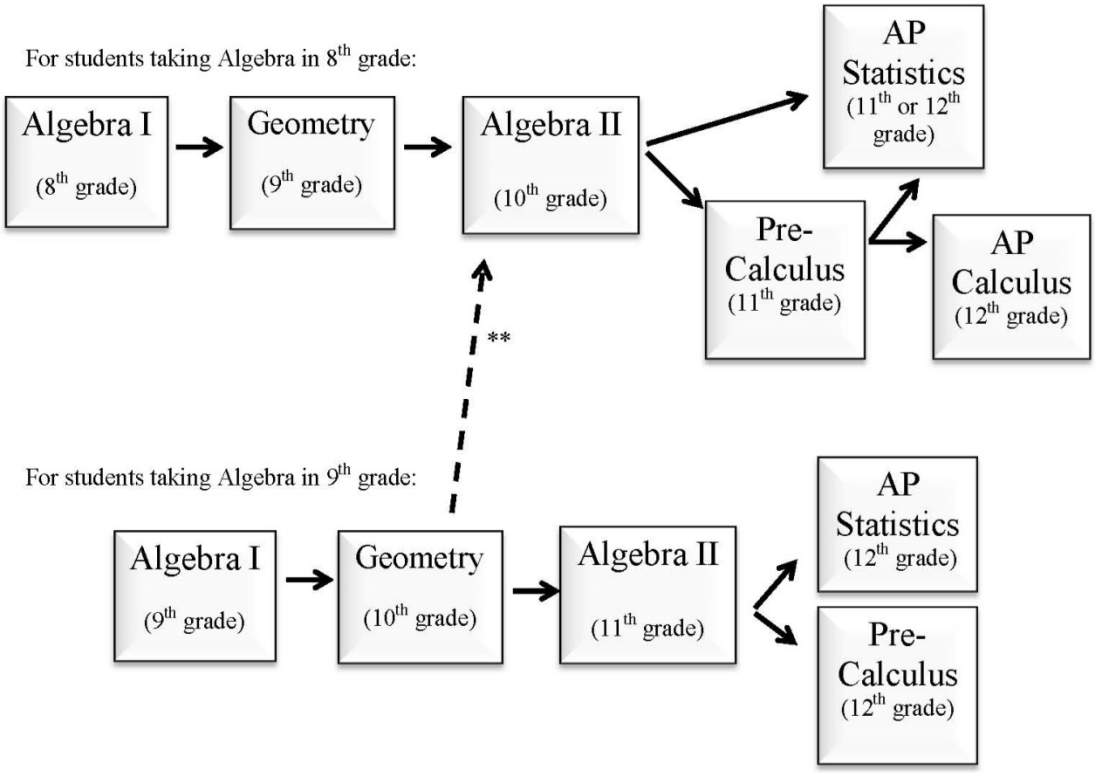
Graduation Requirements
Students must earn three math credits to graduate, but can earn as many as six credits from the following course offerings: Algebra 1A and Algebra 1B *(see below for more information)* or Algebra 1 (one year), Geometry, Algebra 2, Pre-Calculus, AP Statistics and AP Calculus (*See Mathematics Class Progressions on page 13 for more information)*. Students may opt to take the AP Statistics and AP Calculus College Board tests, earning college credit upon successful completion.

Modified Math Program
This program is designed for students with Individual Learning Plans (ILPs) or with other special learning needs. The program is composed of 3 years of mathematics; Foundations of Algebra, Foundations of Geometry, and Applied Mathematics. All incoming students to this program will take Foundations of Algebra first, and it will be offered every year. Foundations of Geometry and Applied Mathematics will be offered alternating years. Students will complete all 3 years of math to meet their Morrison Academy graduation requirements.

Procedure 297 on Class Size states that "Foundations of Algebra, Foundations of Geometry and Applied Mathematics class size may drop below the minimum of five students."

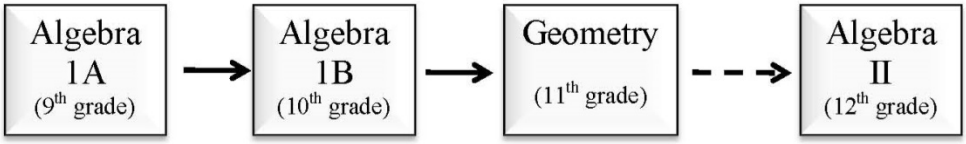
Doubling Up of Math Courses
A student's petition to take Geometry and Algebra 2 concurrently in 10th grade in order to join the accelerated math track can be made through the school counselor. This decision will be made based on student's academic achievement in math, and teacher recommendation. A student's petition to accept summer school credits and /other "fast track" alternatives for taking AP Calculus during their senior year can be made through the high school principal. Students also have the option to complete math courses through an online program, but this decision must be approved by the school counselor prior to enrollment (in accordance with Procedure 202)

Mathematics Class Progressions at Morrison Academy



**Students in this track who want to reach Calculus by their senior year may, upon teacher approval, take Geometry and Algebra II during their sophomore year. Please see the Pacing, Sequence, and Course Offerings section of the Math Curriculum Guide.

For students taking Algebra 1A and 1B:



The first three classes in this track meet the minimum requirement for graduation. However, many universities would like students to take Algebra II before admission.

Typical Progression ⟶

Possible Progression - - - - - ⟶

8th Grade Algebra I: Student Entrance Requirements

Students need to successfully meet the characteristics of Part I and Part II before moving into Part III.

PART I. Minimum Grade Requirement

- At least a 90% in math (cumulative)

PART II. Approval and recommendation of math teachers.

Student Characteristics

- Self-motivated and good study habits

Qualifications of an outstanding math student:

Classwork:
- Participates
- Asks probing questions
- Uses various concepts and methods to solve problems (risk taking)

Homework:
- Completes assignments
- Communicates understanding
- Gives attention to details
- Takes risks - uses known methods to answer a new kind of problem

Tests:
- Expresses answers in complete thoughts
- Works efficiently in appropriate time slated for work
- Gives appropriate amount of information
- Obtains high test scores

NOTE: *The principal may make exceptions after consulting with the Grade 7 Math teacher.*

PART III. Test

Algebra Aptitude Test - grade of 85% or better

PART IV. Final approval

Final placement approval will be given by the principal. Exceptions may be made with principal approval. Acceptance would be conditional upon meeting further requirements.

Overview by Grade/Course

Elementary School

Kindergarten

Kindergarten is the time to start thinking about numbers! The Counting and Cardinality strand takes precedence during this year. More learning time in Kindergarten is devoted to number sense than to other topics. Kindergarten focuses on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space.

Grade 1

First grade continues to focus on the importance of numbers, as we build a strong sense of place value. There are four critical areas of focus this year: (1) developing understanding of and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as repeating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

Grade 2

Second grade is the last year we focus intently on place value. The four critical areas of focus in 2nd grade are (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. Students continue to use a variety of strategies to add and subtract, rather than rely only on the standard algorithm for adding and subtracting. Students also learn about Taiwan money during this year.

Grade 3

In Grade 3, instructional time shifts to a focus on fractions, along with a conceptual understanding of multiplication and division. The critical areas are:(1) developing understanding of and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Grade 4

In Grade 4, we continue to work on ideas involving fractions, multiplication, and division. Students work on multiplying and dividing using a variety of strategies to ensure a solid conceptual understanding of the processes. The key areas are: (1) developing understanding and fluency with multi-digit multiplication, and dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Grade 5

Fifth grade continues to focus on fractions, multiplying, and dividing. Students begin exclusively using the standard algorithm for multiplication, but continue using a variety of conceptual strategies for division. Instructional time focuses on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Middle School

Grade 6

Beginning in 6th grade, focus shifts to rates, ratios, and proportions. In addition, students begin to use the standard algorithm for division exclusively. A strand covering "Expressions and Equations" is introduced, which increases the abstract and algebraic nature of the mathematics. The four critical areas in 6th grade are: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

Grade 7

7th grade continues to focus on ratios, rates, and proportions, as well as algebraic representations. The critical areas are: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

Grade 8

Eighth grade introduces a strand on Functions, preparing students for entry into high school algebra next year. In Grade 8, instructional time focuses on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

High School

Algebra 1

This course builds a solid foundation of algebraic skills. Topics covered include solving linear equations and inequalities. Functions are represented graphically, numerically, symbolically and verbally. Systems of equations and inequalities are created, solved, and the solutions are interpreted. Quadratic equations are factored, graphed and solved, including methods of completing the square and the quadratic formula. Other areas of focus are rational expressions and equations, fitting data to models, comparing linear, quadratic, and exponential functions, and looking at more specialized functions, including absolute value and step functions. Students taking Algebra I in 8th grade must score a B- or higher in the course and on the Algebra Exit Exam.
(See Pacing Guide on page 12 for more information.)

Foundations of Algebra

Foundations of Algebra is for students who are not ready for, or do not need, a fully-paced Algebra 1 course. Topics are selected from the regular Algebra 1 course that are either fundamental or more applicable for students with special needs. Topics covered may include the following: Real Number System (RN), Quantities (Q), Seeing Structure in Expressions (SSE), Creating Equations (CED), Reasoning with Equations and Inequalities in one variable (REI), Interpreting Functions (IF), Building Functions (BF), Linear, Quadratic, and Exponential Models (LE), Interpreting Categorical and Qualitative Data (ID), and Making Inferences and Justifying Conclusions (IC). Students must have completed 8th grade math and have teacher recommendation with Learning Specialist approval. This course is offered every year.

Foundations of Geometry

Foundations of Geometry is for students who are not ready for, or do not need, a fully-paced Geometry course. Topics are selected from the regular Geometry course that are either fundamental or more applicable for students with special needs. Topics covered may include the following: Geometry Essentials (points, lines, planes, and angles), Special Angle Relationships, Parallel and Perpendicular Lines, Congruent Triangles, Similarity, Transformations, and Properties of Circles. Students are encouraged to learn but not required to write proofs. Students must have completed Algebra 1 or Foundations of Algebra and have teacher recommendation with Learning Specialist approval. This course is offered alternating years with Applied Mathematics.

Applied Mathematics

Applied Mathematics is for students who are not ready for, or do not need, Algebra 2. Topics are selected from the Algebra and Geometry courses that relate to Applied Mathematics that are either fundamental or more applicable for students with special needs. Topics covered may include the following: Data Analysis, Probability, Personal Finance, and Practical

Applications. Students must have completed Algebra 1 or Foundations of Algebra and have teacher recommendation with Learning Specialist approval. This course is offered alternating years with Foundations of Geometry.

Geometry

This course in plane and solid geometry emphasizes inductive and deductive reasoning and their applications to geometric figures. Transformations are introduced early, and connections are made to various other topics, such as using dilations to teach the idea of similarity and proportion between figures. Other topics covered include reasoning, congruent and similar polygons, circles, and area and volume measurements. Scientific method, proofs and algebra will be heavily integrated throughout the course as well as a geometry-based computer drawing program.
(*Prerequisite*: Algebra 1)

Algebra 2

This course reviews Algebra 1 with increased emphasis on equations and inequalities, the real number system, functions, relations, graphs of polynomial functions and conic sections, polynomials and algebraic expressions, exponents and numerical computation, and quadratic relations. The content includes systems of equations in three variables, introduction to complex number systems, and study of trigonometric and circular functions.
(*Prerequisite*: Algebra 1, Geometry recommended)

Honors Algebra 2

This course is identical to Algebra 2 except that concepts will be explored in much more detail. Emphasis will be placed on understanding underlying theories and concepts and skills such as critical thinking and problem solving will be stressed. See Algebra 2 description for more information. This is not an AP level course. GPA will be figured on the normal 4.0 scale.
(*Prerequisite*: Algebra 1 in 8th grade, Geometry)

Pre-calculus

Pre-calculus completes the formal study of the elementary functions begun in Algebra 1 and Algebra 2. Students focus on the use of technology, modeling, and problem solving involving data analysis, trigonometric functions, their inverses, polar coordinates, complex numbers, conics, and quadratic relations. Discrete topics include the Principles of Mathematical Induction, the Binomial Theorem, and sequences and series. Students are required to have a graphing calculator. See teacher for details.
(*Prerequisites*: Geometry and a grade of B- (80%) or above in Algebra 2 or teacher recommendation)

AP Statistics

This course is the equivalent of a one-semester, introductory, non-calculus-based college course in statistics. Most students who take AP Statistics will also take college courses in the physical, biological, and social sciences, and the AP course will provide an excellent foundation for subsequent college work. Students engage in four broad conceptual themes: (1) Exploring data and describing patterns, (2) Sampling and experimenting by planning and conducting a study, (3) Exploring random phenomena using probability and simulation, and (4) Estimating population parameters and testing hypotheses.
[www.collegeboard.org] Students are required to have a graphing calculator. See teacher for details. See AP courses policies in Course Handbook.
(*Prerequisites*: Geometry and a grade of B- (80%) or above in Algebra 2 or teacher recommendation)
Maximum of 18 students per section.

AP Calculus AB

This course is designed for those students who have shown high achievement in previous math courses and desire further training in advanced mathematics. It will prepare students for the Advanced Placement Calculus AB exam and college level mathematics equivalent to a first semester college-calculus course. The course will focus both on differential and integral calculus. The student is required to purchase a graphing calculator. See teacher for details. See AP courses policies in Course Handbook.
(*Prerequisite*: B- or better in Pre-Calculus and/or teacher recommendation).
Maximum of 18 students per section.

Kindergarten Benchmarks

Counting and Cardinality K.CC

Know number names and the count sequence.

1. I can count to 100 by ones and by tens
2. I can count forward from any given number within 100
3. I can write numbers 0-20 and represent a group of objects with a written numeral.

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
    a. When counting objects, I can say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
    b. I understand that the last number states the number of object being counted regardless of the order in which they are counted. The arrangement and order of the objects does not affect the number of objects.
    c. I understand that each successive number shows a quantity that is one larger.
5. I can answer "how many" are in a set within 20, in a variety of configurations. Given a number from 1-20, I can count out that many objects.

Compare numbers.

6. I can identify if the number of objects is greater than, less than, or equal to the number of objects in another group (by using matching and counting strategies)
7. I can compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. I can show addition and subtraction using a variety of methods (i.e.: objects, fingers, mental images, drawings, acting out, expression, verbal explanations, or equations)
2. I can solve addition and subtraction word problems within 10 and use objects or drawings to represent the problem
3. I can decompose numbers less than or equal to 10 into pairs in more than one way (objects, drawings, or equations).
4. For any number from 1 to 9, I can find the number that makes 10 when added to the given number (i.e.: by using objects or drawings, and record the answer with a drawing or equation)
5. I can add and subtract fluently within 5

Number and Operations in Base Ten K.NBT

Work with numbers 11–19 to gain foundations for place value.

1. I can compose and decompose numbers from 11-19 into ten ones and some extras by using objects or drawings. I can record these compositions by using a drawing or equation.

Measurement and Data K.MD

Describe and compare measurable attributes.

1. I can describe several measurable attributes of objects, such as length or weight.

Describe several measurable attributes of a single object.

2. I can compare two objects with a measurable attribute in common (example: height of two children and determine who is taller or shorter).

Classify objects and count the number of objects in each category.

3. I can sort objects into given categories; count the numbers of objects in each category and sort the categories by count.

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Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. I can describe objects in the environment using names of shapes and determine their relative location (i.e.: above, below, beside, in front of, behind, next to)
2. I can correctly name shapes regardless of orientation or size
3. I can identify shapes as two dimensional (lying in a plane, "flat") or three dimensional shapes ("solid")

Analyze, compare, create, and compose shapes.

4. I can analyze and compare 2D and 3D shapes in different sizes and orientations and describe their similarities and differences (number of sides and vertices / corners, and other attributes)
5. I can draw, model, and build shapes using a variety of materials.
6. I can use simple shapes to compose a larger shape (i.e.: two triangles can make a rectangle).

Grade 1 Benchmarks

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. I can use addition and subtraction within 20 to solve word problems using a variety of methods, with the unknown quantity in all positions (with a symbol for the unknown number to represent the problem)
2. I can solve word problems that call for adding three whole numbers whose sum is less than or equal to 20 using a variety of methods.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. I can use properties of operations to add and subtract (i.e. commutative and associative properties).
4. I understand subtraction as an unknown addend problem.

Add and subtract within 20.

5. I can relate counting to addition and subtraction
6. I can use a variety of strategies to add and subtract within 20 and fluently within 10

Work with addition and subtraction equations.

7. I demonstrate understanding of the equal sign and can determine whether an equation is true or false
8. I can determine the unknown whole number in addition and subtraction equations

Number and Operations in Base Ten 1.NBT

Extend the counting sequence.

1. I can count to 120, starting at any number. I can read and write numbers through 120, and represent a number of objects with a written numeral.

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. I demonstrate understanding that ten ones equal one tens
   b. I understand that numbers from 11 to 19 are made up of one ten and some extra ones
   c. I understand that the numbers 10, 20, 30,... refer to one, two, three,… tens and zero ones
3. I can compare two digit numbers using the symbols >, =, <

Use place value understanding and properties of operations to add and subtract.

4. I can use my understanding of place value and properties of operations to add within 100, using a variety of visual models and strategies; including cases where it is necessary to compose a ten.
5. I can find ten more or ten less of a number without counting and explain or model my reasoning

6. I can subtract multiples of ten in the range of 10-90 from multiples of 10 in the range of 10-90 (positive or zero differences) using a variety of visual models and strategies and demonstrate the reasoning

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1. I can order and compare lengths of three objects. I can compare the lengths of two objects indirectly by using a third object.
2. I can express an object's length as a whole number of length units. I understand that the length measurement of an object is the number of same-size length units with no gaps or overlaps.

Tell and write time.

3. I can tell and write time in hours and half hours using analog and digital clocks

Represent and interpret data.

4. I can organize, represent, and interpret data with up to three categories. I can ask and answer questions about the total amount of data and compare data between categories

Geometry 1.G

Reason with shapes and their attributes.

1. I can distinguish between defining attributes (e.g.: triangles are closed and three sided) v. non-defining attributes (e.g.: color, orientation, overall size) and build and draw shapes with these attributes
2. I can use 2D shapes and 3D shapes to create composite shapes, and compose new shapes from the composite shape
3. I can partition circles and rectangles into two or four equal shares and describe them using the words "halves, fourths, quarters." I can describe the whole as two of, or four of the shares. I understand that decomposing into more equal shares creates smaller shares.

Grade 2 Benchmarks

Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. I can use addition and subtraction within 100 to solve one and two-step word problems using a variety of strategies

Add and subtract within 20.

2. I can fluently add within 20. By the end of 2nd grade I know from memory the sums of all one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. I can determine if a group has an odd or even number of members using a variety of strategies, and write an equation to express an even number as the sum of two equal addends.
4. I can use skip counting and repeated addition to interpret rectangular arrays (up to 5 rows by 5 columns), and express the total in an equation as the sum of equal addends.

Number and Operations in Base Ten 2.NBT

Understand place value.

1. I can identify the place value for digits in a 3-digit number
    a. I can describe and represent 100 as a bundle of ten tens
    b. I can describe and represent multiples of 100 (up to 1000) as bundles of 100
2. I can count up to 1000 and skip count by 5's, 10's, and 100's
3. I can read and write numbers up to 1000 using standard form, expanded form, and word form
4. I can compare two 3 digit numbers using >, <, or =.

Use place value understanding and properties of operations to add and subtract.

5. I can add and subtract within 100, with and without regrouping, using a variety of strategies.
6. I can add up to four two-digit numbers using a variety of strategies.
7. I can add and subtract up to 1000 using a variety of strategies; and explain how numbers of the same place value need to be added or subtracted together, and how re-grouping works.
8. I can mentally add and subtract 10 or 100 to a given number within 1000
9. I can explain why addition and subtraction strategies work using ideas of place value and properties of operations.

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

1. I can select and use appropriate tools to measure the length of an object
2. I can measure the length of an object twice using different units and describe how the two measurements are related.
3. I can estimate lengths using inches, feet, centimeters, and meters
4. I can measure to determine how much longer one object is than another using a standard length unit.

Relate addition and subtraction to length.

5. I can use a variety of strategies to solve addition and subtraction word problems within 100 that involving lengths in the same unit
6. I can represent whole numbers, sums, and differences within 100 on a number line diagram

Work with time and money.

7. I can tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
8. I can identify, count, compare, and solve word problems involving dollar bills, quarters, dimes, nickels, and pennies; using dollar and cent symbols appropriately

Represent and interpret data.

9. I can generate data by measuring lengths of several objects and graph it using a line plot (with whole number units).
10. I can create graphs (picture, bar, and line plot) with up to four categories; and I can interpret the graphs to solve simple word problems involving that data.

Geometry 2.G

Reason with shapes and their attributes.

1. I can recognize and draw 2-D and 3-D shapes given specific attributes. I can identify quadrilaterals, pentagons, hexagons, and cubes.
2. I can find the area of a rectangle by dividing it into rows and columns of equal sized squares and counting to find the total.
3. I can split circles and rectangles into 2, 3, or 4 equal parts, and use fraction terms to identify halves, thirds, quarters, and wholes. I can also recognize that equal shares of identical wholes are not necessarily the same shape.

Grade 3 Benchmarks

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. I can interpret multiplication problems as the total number of objects when you have a given number of groups each containing a given number of objects. I can describe a context in which this is applicable.
2. I can interpret quotients of whole numbers as the number of objects in each share when a total number of objects are separated equally into a number of shares. I can describe a context in which this is applicable.
3. I can use a variety of strategies solve multiplication and division word problems (within 100)
4. I can use fact families to determine an unknown whole number in a multiplication or division equation (within 100)

Understand properties of multiplication and the relationship between multiplication and division.

5. I can identify and apply properties of operations as strategies to multiply and divide (commutative, associative, distributive).
6. I can use multiplication facts to solve a division problem with an unknown.

Multiply and divide within 100.

7. I can fluently multiply and divide within 100. I know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. I can solve 2-step word problems in all four operations using a variable for the unknown, and check reasonableness of answer using estimation strategies (including rounding).
9. I can identify and explain arithmetic patterns in addition and multiplication (including the multiplication table) and can explain the patterns using properties of operations.

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. I can use place-value understanding to round whole numbers to the nearest 10 or 100
2. I can fluently add and subtract numbers within 1000 (up to 3 digit by 3 digit) including regrouping using a variety of strategies.
3. I can multiply one digit whole numbers by multiples of 10 (10-90) using strategies based on place value and properties of operations.

Number and Operations—Fractions 3.NF

Develop understanding of fractions as numbers.

1. I understand that the fraction "1/b" is one part when a whole is broken into b equal parts, and understand a fraction a/b is made up of "a" equal parts of size "1/b"
2. I can represent fractions on a number line diagram.
    a. I can represent a fraction on a number line by splitting the interval from 0 to 1 into "b" equal parts, and locate "1/b" on the number line.
    b. I can represent a fraction "a/b" on a number line by marking off equal segments of "1/b" from 0, and counting "a" segments from 0.
3. I can compare fractions by reasoning about their size and explain equivalence of common fractions (halves, thirds, fourths, fifths, eighths, tenths)
    a. I can demonstrate that two fractions are equivalent if they are the same size or are at the same point on a number line.
    b. I can recognize and generate common equivalent fractions, and use a visual fraction model to explain why they are the same.
    c. I can express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
    d. I can compare two fractions with like numerators or like denominators, in reference to the same whole, using >, <, = symbols, and justify my conclusions.

Measurement and Data 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. I can tell and write time to the nearest minute and measure time intervals (elapsed time) in minutes; and solve word problems involving elapsed time in minutes.
2. I can estimate and find the volume and weight of various figures using the metric system (grams, kilograms, and liters); and solve one-step word problems involving volume or weight

Represent and interpret data.

3. I can create, interpret, and draw conclusions from pictographs and bar graphs.
4. I can generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch, and graph it using a line plot.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. I can recognize area as an attribute of plane figures and understand concepts of area measurement.
    a. I can explain that a square with side length 1 unit is "unit square" and can be used to measure area.
    b. I can use a "unit square" to tile a plane figure to measure area, where tiling it with n unit squares means it has an area of n square units.
6. I can measure areas using unit squares of various units (square cm, square m, square ft., and improvised units)

7. I understand how area is related to the operations of addition and multiplication
    a. I can find the area of a rectangle by tiling unit squares, show that this is the same as I would get by multiplying side lengths
    b. I can use multiplication to solve word problems involving area of rectangles.
    c. I can use tiling to show that the area of a rectangle with side lengths "a" and "b+c" is the same as the sum of the area of two rectangles a by b and a by c. I can use this model to represent the distributive property
    d. I can find the area of composite shapes by splitting it into non-overlapping rectangles and adding the areas of these parts; and solve word problems involving this concept.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. I can find the perimeters of polygons and solve for an unknown side length; and solve word problems involving this concept.

Geometry 3.G

Reason with shapes and their attributes.

1. I know the attributes of shapes, understand that shapes in different categories may share attributes. I know that rhombuses, rectangles, and squares are all examples of quadrilaterals, and can draw examples of other quadrilaterals that do not belong to any of these subcategories.
2. I can split shapes into equal shares, and express the area of each share using the words halves, thirds, fourths, etc.

Grade 4 Benchmarks

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

1. I understand multiplication equations as comparisons, (35 is 5 times as many as 7 and also 7 times as many as 5), and can represent verbal statements of comparison as multiplication equations.
2. I can multiply or divide to solve word problems involving multiplicative comparison, using a variety of strategies.
3. I can apply a variety of appropriate strategies (including creating an equation with a variable) to solve multi-step word problems with whole numbers using the four operations and check the reasonableness of answers

Gain familiarity with factors and multiples.

4. I can identify multiples, factor pairs, and classify numbers as prime or composite (1-100)

Generate and analyze patterns.

5. I can generate a number or shape pattern that follows a given rule (including repeating and growing patterns), and identify features of the pattern.

Number and Operations in Base Ten2 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. I understand that in a multi-digit whole number, a digit in one place represents 10 times what is represented in the place to its right
2. I can read and write numbers using base-ten numerals, number names, and expanded form, as well as compare two numbers using <, >, or =.
3. I can round whole numbers to any place using place value understanding

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. I can fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. I can multiply multi-digit numbers (four-digits by one-digit and two-digits by two-digits) using a variety of methods, and explain my calculations using equations, arrays, and/or area models.
6. I can divide multi-digit numbers with and without remainders (up to a four-digit dividend by one-digit divisor) using a variety of strategies, and explain my calculations using equations, arrays, and/or area models.

Number and Operations—Fractions3 4.NF

Extend understanding of fraction equivalence and ordering.

1. I can explain why a/b is equivalent to (n x a) / (n x b) using visual fraction models. I can use this principle to recognize and generate equivalent fractions.
2. I can compare two fractions with different numerators and different denominators using a variety of methods, record the results using <, =, or >, and justify my conclusions.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. I understand that a fraction a/b with a>1 as a sum of fractions 1/b
    a. I can add and subtract fractions by joining and separating parts of the same whole.
    b. I can decompose a fraction into a sum of fractions with the same denominator in more than one way (ex. 3/8 = 1/8+1/8+1/8 or 3/8=1/8+2/8) and justify my reasons.
    c. I can add and subtract mixed numbers with like denominators using a variety of methods
    d. I can solve word problems involving adding and subtracting of fractions with like denominators using a variety of methods
4. I can apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
    a. I understand a fraction a/b as a product of 1/b and a whole number (i.e.: 5/4 = 5 x 1/4)
    b. I can multiply a fraction (a/b) by a whole number (i.e.: n x (a/b) = (n x a)/b)
    c. I can solve word problems involving multiplication of a fraction by a whole number using a variety of methods

Understand decimal notation for fractions, and compare decimal fractions.

5. I can add fractions with a denominator of 10 and 100 by obtaining a common denominator (i.e.: 3/10 + 4/100)
6. I can convert from fractions to decimals for fractions with denominators 10 or 100 (tenths or hundredths)
7. I can compare decimals to the hundredth place using <, >, or =, and justify my conclusions

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. I know relative sizes of measurement units and can convert from a larger unit to a smaller unit within customary or metric units (k,m,cm ; kg,g ; lb,oz ; l,ml ; hr,min,sec)
2. I can use the four operations to solve problems involving measurement units (i.e.: distances, intervals of time, liquid volumes, masses of objects, and money) including problems involving simple fractions or decimals or requiring conversions
3. I can solve real-world and mathematical problems using the area and perimeter formulas.

Represent and interpret data.

4. I can make a line plot to display a set of measurements in fractions of a unit (i.e.: 1/2 cm, 1/4 cm, 1/8 cm) and solve word problems involving addition and subtraction of fractions using this data.

Geometric measurement: understand concepts of angle and measure angles.

5. I recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.
    a. I can understand that an angle is measured with reference to a circle (360 degrees), and what is meant by the terms "angle" and "degree". An angle that turns 1/360 of the circle is called a one degree angle.
    b. I understand that an angle that turns through n one-degree angles is said to have an angle measure of n degrees.
6. I can measure and sketch angles to a specified measure (whole number degrees) using a protractor
7. I can solve real world and mathematical problems involving addition and subtraction of angles (i.e.: finding an unknown angle in a diagram), knowing that the angle measure of the whole is the sum of the angle measures of the parts.

Geometry 4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. I can identify and draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines
2. I can classify two-dimensional shapes from parallel or perpendicular lines and angle size (i.e.: trapezoid, parallelogram, rectangle, square, rhombus ; acute, obtuse, and right triangles)
3. I can understand, identify, and draw lines of symmetry for a two-dimensional figure

Grade 5 Benchmarks

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

1. I can use parentheses/brackets/braces in expressions, and evaluate expressions with these symbols.
2. I can write simple expressions that record calculations with numbers, and interpret these expressions without evaluating them. (ex. Know that 3(5 + 4) mea

Analyze patterns and relationships.

3. I can generate a pattern when given a rule. I can also do this for two separate patterns at once, form ordered pairs with corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

Number and Operations in Base Ten 5.NBT

Understand the place value system.

1. I understand the place value system (i.e.: recognize that a digit in a particular place is 10 times as much as the place to its right, and 1/10 of what the place to its left)
2. I can explain the pattern in zeros or decimal point placement when whole numbers or decimals are multiplied or divided by powers of ten. I can use who number exponents to name powers of 10.
3. I can read, write, and compare decimals to the thousandths place
    a. I can read and write decimals to the thousandths place using numbers, words or expanded form.
    b. I can use my knowledge of place value to compare decimals to the thousandths place using <. =, or >.
4. I can use my knowledge of place value to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. I can fluently multiply multi-digit numbers using the standard algorithm.
6. I can divide whole numbers (up to 4-digit dividends and 2-digit divisors) using a variety of strategies and models. I can illustrate and explain my calculations using equations, arrays, and/or area models.
7. I can add, subtract, multiply, and divide decimals to the hundredths place using a variety of strategies. I can relate my strategy to a written method and explain my reasoning.

Number and Operations—Fractions 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. I can add and subtract fractions with unlike denominators (including mixed numbers) by finding equivalent fractions with like denominators
2. I can solve word problems involving addition and subtraction of fractions (including like and unlike denominators) using a variety of strategies, and estimate to check the reasonableness of the answer

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. I understand that a fraction is a form of writing division (of the numerator by the denominator), and can solve word problems with division of whole numbers where the answer is a fraction of mixed number using a variety of methods.
4. I can multiply two fractions (or a fraction with a whole number).
    a. I understand that the process of multiplying a fraction by a whole number is the same as repeated addition of the fraction, and see that it is equivalent to multiplying the numerator by the whole number, then dividing by the denominator..
    b. I can find the area of a rectangle with fraction side lengths (by multiplying side lengths and by modeling with tiles)
5. I can interpret multiplication as a method of scaling (re-sizing)
    a. I can compare the size of a product (result) to the size of one factor based on the size of the other factor without performing the multiplication
    b. I can explain how multiplying a number by a fraction greater than one results in a greater number, and how multiplying a number by a fraction less than one results in a lesser number
6. I can solve word problems involving multiplication of fractions or mixed numbers using visual models or equations to represent the problem.
7. I can divide unit fractions by whole numbers, and vice versa
    a. I can divide a unit fraction by a non-zero whole number, and create a story context for the equation
    b. I can divide a whole number by a unit fraction, and create a story context for the equation
    c. I can solve word problems involving division of unit fractions by natural numbers and division of whole numbers by unit fractions using visual fraction models and equations to represent the problem.

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

1. I can convert standard measurement units within a given measurement system and use these conversions to solve multi-step word problems

Represent and interpret data.

2. I can make a line plot to display a data set that includes fractions, and solve problems that involve the information presented in the line plot.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. I understand that volume is an attribute of solid figures, and how volume is measured.
    a. I understand how a cube with side length of 1 unit is called a "unit cube" and can be used to measure volume
    b. I understand a solid figure filled with "n" unit cubes has a volume of "n" cubic units
4. I can measure volume by counting unit cubes (cubic cm, cubic in, cubic ft., etc.)
5. I can relate volume to multiplication and addition to solve real-world and mathematical problems.
    a. I can find the volume of a rectangular prism using unit cubes, and show that it is the same as is found by multiplying length x width x height.
    b. I can use the formulas V=lwh and V=Bh to find the volume of right rectangular prisms in real-world and mathematical problems.
    c. I understand that I can find the volume of a solid figure formed by two rectangular prisms by adding the volumes of the two parts together, and use this knowledge to solve real-world problems.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

1. I understand the make-up of a coordinate (Cartesian) plane, (including x-axis, y-axis, ordered pairs, origin, x-coordinate, and y-coordinate) and can plot coordinates on the plane
2. I can use the first quadrant to represent real-world and mathematical problems by graphing points, and interpret coordinate values in the context of the problem.

Classify two-dimensional figures into categories based on their properties.

3. I understand that the various attributes of a category of 2-dimensional figures are shared by all figures in subcategories of that category.
4. I can classify 2-D figures in a hierarchy based on their properties.

Grade 6 Benchmarks

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
    a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
    b. Solve unit rate problems including those involving unit pricing and constant speed.
    c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
    d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System 6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions

Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
    a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
    b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
    c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
7. Understand ordering and absolute value of rational numbers.
    a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
    b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.
    c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
    d. Distinguish comparisons of absolute value from statements about order.
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
    a. Write expressions that record operations with numbers and with letters standing for numbers.
    b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
    c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
3. Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x

+ 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.

4. Identify when two expressions are equivalent

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.
8. Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Geometry 6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability 6.SP

Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
    a. Reporting the number of observations.
    b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
    d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Grade 7 Benchmarks

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
2. Recognize and represent proportional relationships between quantities.
    a. Decide whether two quantities are in a proportional relationship, (test equivalent ratios, graph in a coordinate plane to see if it goes through origin, etc.)
    b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
    c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
    d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems.

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
    a. Describe situations in which opposite quantities combine to make 0.
    b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
    c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
    d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
    a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such

as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.

8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions [1]

<u>Expressions and Equations 7.EE</u>

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
    a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

[1] These benchmarks are introduced to all students in 7th grade and repeated in 8th grade (to prepare all 7th grade

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

8.EE.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.
8.EE.2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
8.EE.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
8.EE.4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.[2]

Geometry 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

[2] These benchmarks are introduced to all students in 7th grade and repeated in 8th grade (to prepare all 7th grade students regardless of which math course they take the following year).

Statistics and Probability 7.SP

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
    a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
    b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
8. Find probabilities of compound events using lists, tables, tree diagrams, and simulation.
    a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
    b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
    c. Design and use a simulation to generate frequencies for compound events.

Grade 8 Benchmarks

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions

Expressions and Equations 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
   c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Functions 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and

translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Algebra I Benchmarks

The Real Number System N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
2. Rewrite (simplify) expressions involving radicals and rational exponents using the properties of exponents.

   8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
   8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.[3]

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions[4]

1. Interpret expressions that represent a quantity in terms of its context
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

2. Use the structure of an expression to identify ways to rewrite it.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   a. Factor a quadratic expression to reveal the zeros of the function it defines.

[3] These benchmarks are to be taught to students taking Algebra I during their 8th grade year. (Students in 9th grade Algebra I will have already covered these benchmarks in their 8th grade math class.)
[4] Algebra I is limited to linear, quadratic, and exponential expressions

c. Use the properties of exponents to transform expressions for exponential functions.

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.[5]

Creating Equations A-CED[6]

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities[7], and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning with Equations and Inequalities A-REI

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions[8] and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

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[5] For Algebra I, focus on adding and multiplying polynomial expressions, factoring or expanding polynomial expressions to identify and collect like terms, applying the distributive property.
[6] Create linear, quadratic, and exponential (with integer domain) equations in Algebra I.
[7] Equations and inequalities in this standard should be limited to linear.
[8] It is sufficient in Algebra I to recognize when roots are not real; writing complex roots is included in Algebra II.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.[9]

7. Solve a simple system consisting of a linear equation and a quadratic[10] equation in two variables algebraically and graphically.

Represent and solve equations and inequalities[11] graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions
12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

MA.13. Solve radical equations in one variable. (limited to square roots)[12]

Interpreting Functions F-IF

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

[9] These benchmarks are to be taught to students taking Algebra I during their 8th grade year. (Students in 9th grade Algebra I will have already covered these benchmarks in their 8th grade math class.)

[10] Algebra I does not include the study of conic equations; include quadratic equations typically included in Algebra I.

[11] In Algebra I, functions are limited to linear, absolute value, and exponential functions for this cluster.

[12] "MA" designates a Morrison added benchmark on solving radical equations.

Interpret functions[13] that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions[14] using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root15, and piecewise-defined functions, including step functions and absolute value functions.
   e. Graph exponential and logarithmic16 functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude[17].
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions[18] F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context
   b. Combine standard function types using arithmetic operations.

[13] Limit to interpreting linear, quadratic, and exponential functions.
[14] In Algebra I, only linear, exponential, quadratic, absolute value, step, and piecewise functions are included in this cluster
[15] Graphing square root and cube root functions is included in Algebra II.
[16] In Algebra I it is sufficient to graph exponential functions showing intercepts
[17] Showing end behavior of exponential functions and graphing logarithmic and trigonometric functions is not part of Algebra I.
[18] Functions are limited to linear, quadratic, and exponential in Algebra I.

Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Linear, Quadratic, and exponential models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).[19]
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential[20] function in terms of a context.

Expressing Geometric Properties with Equations G-GPE

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

[19] Arithmetic and geometric sequences are not covered until Algebra II.

[20] Limit exponential function to the form $f(x) = b^x + k$.

Summarize, represent, and interpret data on two categorical and quantitative variables[21]

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

Investigate patterns of association in bivariate data.

8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. [22]

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

[21] Linear focus; discuss as a general principle in Algebra I.

[22] These benchmarks are to be taught to students taking Algebra I during their 8th grade year. (Students in 9th grade Algebra I will have already covered these benchmarks in their 8th grade math class.)

Geometry Benchmarks

Congruence G-CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems[23]

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
10. Prove theorems about triangles.
11. Prove theorems about parallelograms.

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

[23] Proving the converse of theorems should be included when appropriate.

Similarity, Right Triangles, and Trigonometry G-SRT

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

4. Prove similarity theorems about triangles.
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Circles G-C

Understand and apply theorems about circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically.
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Geometric Measurement and Dimension G-GMD

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry G-MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts.

Algebra II Benchmarks

The Real Number System N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
2. Rewrite (simplify) expressions involving radicals and rational exponents using the properties of exponents.

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

The Complex Number System N-CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions[24]

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.
2. Use the structure of an expression to identify ways to rewrite it.

[24] Previously covered in Algebra I, but limited to linear, quadratic, and exponential expressions.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Write expressions in equivalent forms to solve problems

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.[25]

Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships.
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations A-CED[26]

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

[25] Previously taught in Algebra I, with a focus on adding and multiplying polynomial expressions, factoring or expanding polynomial expressions to identify and collect like terms, applying the distributive property.
[26] Previously taught in Algebra I, limited to linear, quadratic, and exponential (with integer domain) equations in Algebra I.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities[27], and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
    b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions[28] and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Represent and solve equations and inequalities[29] graphically

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions

Interpreting Functions F-IF

Understand the concept of a function and use function notation

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

[27] Previously taught in Algebra I, but limited to linear equations and inequalities.
[28] Previously taught in Algebra I, but limited to recognizing when roots are not real.
[29] Previously taught in Algebra I, but limited to linear, absolute value, and exponential functions for this cluster.

Interpret functions[30] that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions[31] using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions F-BF[32]

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context
    b. Combine standard function types using arithmetic operations.
    c. Compose functions.
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

[30] Previously taught in Algebra I, but limited to interpreting linear, quadratic, and exponential functions.
[31] Previously taught in Algebra I, but limited to linear, exponential, quadratic, absolute value, step, and piecewise functions in this cluster.
[32] Previously covered in Algebra I, but limited to linear, quadratic, and exponential.

Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse
   b. Verify by composition that one function is the inverse of another.
   c. .Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. Produce an invertible function from a non-invertible function by restricting the domain.
5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems in involving logarithms and exponents.

Linear, Quadratic, and exponential models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Construct and compare linear, quadratic, and exponential models and solve problems

4. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Model periodic phenomena with trigonometric functions

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context

Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin 2 (θ) + cos 2 (θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.

Similarity, Right Triangles, and Trigonometry G-SRT

Apply trigonometry to general triangles

9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove[33] the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section

2. Derive the equation of a parabola given a focus and directrix.
3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

[33] Proof of these laws will be covered in Pre-Calculus

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Using Probability to Make Decisions S-MD

Use probability to evaluate outcomes of decisions

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts

Pre-Calculus Benchmarks

Trigonometry PC-T

1. I know the definition of an angle and how to measure it, in both degrees and radians. I can convert between degrees and radians and apply it various situations.
2. I understand the unit circle and know the definitions of six trigonometric functions and their inverses. I can compute, by hand, the values of the trigonometric functions and their inverse at various standard angles on the unit circle.
3. I can solve right triangles using the concepts of trigonometry.
4. I can graph and manipulate the six basic trigonometric functions and their inverses.
5. I can use the general form of sine and cosine functions to solve a variety of real-life problems including applications of harmonic motion.
6. I can simplify and solve trigonometric equations using basic reciprocal, product, and Pythagorean identities.
7. I can use a variety of trigonometric formulas to prove and/or simplify other trigonometric identities, as well as solve trigonometric equations.
8. I can state and prove the law of sines and the law of cosines and apply those laws to solve problems.

Linear Algebra PC-LA

1. I can solve a system of equations using a variety of methods including interpreting systems of equations as coefficient in a matrix.
2. I can perform and apply the operations of addition, subtraction, multiplication, scalar multiplication, and inversion, on matrices and vectors.
3. I can compute the determinants of 2×2 and 3×3 matrix and am familiar with applications of the determinant as well as its use in finding the inverse of a square matrix.
4. I can represent two- and three-dimensional vectors both geometrically and in component form. I can perform basic vector operations.
5. I can compute the scalar (dot) product and (cross) product of two vectors in n-dimensional space and can identify that vectors with a dot product of zero are perpendicular.

Discrete Mathematics PC-DM

1. I understand and can apply factorial expressions and summation notation.
2. I can explain the difference between sequences and series. I can extend arithmetic and geometric sequences as well as compute their series and if the sequence converges.
3. I can give proofs of various formulas by using the technique of mathematical induction.
4. I know and understand the notion of independent and dependent events, and can use fundamental counting principles to compute combinations, permutations, and probabilities.

5. I know and can determine the mean, median, mode and the standard deviation of a distribution of data.

Analytic Geometry PC-AG

1. I understand equations defined parametrically and how to use, convert, and apply them. I can use a graphing calculator in parametric mode.
2. I understand how to use, convert, and apply polar coordinates in the plane. I can use a graphing calculator in polar mode.

Elementary Functions PC-EF

1. I can solve quadratic equations by a variety of methods in the complex number system. I am able to use these techniques to solve word problems.
2. I know the fundamental theorem of algebra and can sketch graphs of higher degree polynomials and can perform operations on them.
3. I can add, subtract, multiply, divide, reduce, graph, and evaluate rational expressions with monomial and polynomial denominators.
4. I understand and can graph exponential functions, and use these functions to solve real-life problems including ones involving exponential growth and decay.
5. I understand and can graph logarithmic functions, and can use properties of logarithms to simplify and solve logarithmic expressions. I can translate between logarithms in any base.

Limits PC-L

1. I understand the definition of a limit and can use a variety of techniques to evaluate a limit if it exists.
2. I am familiar with infinite limits and can apply them to sequences, summations, and other applications.

High School - AP Statistics Benchmarks *(from the AP Board)*

Exploring Data: Describing patterns and departures from patterns S-ED

a. Constructing and interpreting graphical displays of distributions of univariate data.
b. Summarizing distributions of univariate data
c. Comparing distributions of univariate data
d. Exploring bivariate data
e. Exploring categorical data

Sampling and Experimentation: Planning and conducting a study S-SE

a. Overview of methods of data collection
b. Planning and conducting surveys
c. Planning and conducting experiments
d. Generalizability of results and types of conclusions that can be drawn from observational studies, experiments and surveys

Anticipating Patterns: Exploring random phenomena using probability and Simulation S-AP

a. Probability
b. Combining independent random variables
c. The normal distribution
d. Sampling distributions

Statistical Inference: Estimating population parameters and testing hypotheses S-SI

a. Estimation
b. Tests of significance

High School - AP Calculus Benchmarks *(from the AP Board)*

Functions, Graphs, and Limits C-FGL

a. Analysis of graphs
b. Limits of functions (including one-sided limits)
c. Asymptotic and unbounded behavior
d. Continuity as a property of functions

Derivatives C-D

a. Concept of the derivative
b. Derivative at a point
c. Derivative as a function
d. Second derivatives
e. Applications of derivatives
f. Computation of derivatives

Integrals C-I

a. Interpretations and properties of definite integrals
b. Applications of integrals
c. Fundamental Theorem of Calculus
d. Techniques of anti-differentiation
e. Applications of anti-differentiation
f. Numerical approximations to definite integrals

Appendix A: Biblical Principles for Math

God

A. God's truth is universal, eternal, and absolute; we can better appreciate these principles through the study of mathematics.
B. Mathematics is an integral part of creation. Mathematical studies reveal the structure of the universe and thus reveal the character and attributes of God.
C. The mathematical complexity, harmony and precision of the created order demonstrate the infinite wisdom of God.

Creation

A. Creation is logical, ordered, and consistent; we understand this through mathematics when we work with numbers, equations, and formulas.
B. God gives man the responsibility to subdue and rule over the earth (Genesis 1:28), which can be better accomplished through discovery and innovation of mathematical principles.

Mankind

A. Man and woman are created in the likeness of God, and are given the ability to understand the mathematical processes and patterns apparent in creation.
B. The ability to see and describe the world in mathematical constructs is a gift of God to man.

Moral Order

A. As we apply mathematical principles (i.e.: finances, practical sciences, etc.), we have the moral responsibility to use these in a way that glorifies God and furthers His kingdom.
B. The study of mathematics helps students develop an appreciation for correctness of procedure, accuracy, and perseverance.

Purpose

A. Through gaining a mastery of mathematical concepts, we can become better equipped to accomplish the purpose and calling God has intended for each of us.